Poincaré limit cycles and the theory of oscillations[1]

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There exist various mechanisms which can generate sustained oscillations as a result of non-periodic energy sources. Nevertheless, there is currently no sufficiently rigorous and general theory of such self-oscillations. Meanwhile we have an adequate mathematical model, created quite outside the theory of oscillations, which allows us to take a general view of all such processes for the case of one degree of freedom. The approach is Poincaré's theory of 'limit cycles'.

The equation of motion of the system under consideration, having one degree of freedom and no explicit time variance, is a non-linear equation of the form:

$$\frac{d^2x}{dt^2} = F\left(x, \frac{dx}{dt}\right) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (1)$$

Let

$$\frac{dx}{dt} = y \, , \; \frac{d^2x}{dt^2} = y\frac{dy}{dx}$$

We can write this equation in the form:

$$\frac{dy}{dx} = \frac{F(x,y)}{y} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (2)$$

We shall assume the variables x and y to be real, and investigate the behaviour of the integral curves of this equation in the x, y phase plane.

It is easy to see that the periodic solutions of equation (1) correspond to closed integral curves in the xy plane.

Apart from certain conditions on $F(x, dx/dt)$, these closed solutions of equation (2) can be of two types only: either so-called 'central' solutions or 'limit cycles'.

'Central' solutions correspond to ideal motion – for, example, the conservative motion of classical mechanics. If equation (2) possesses a limit cycle – that is, if all the curves in a given region tend towards the limit cycle as $t \to \pm\infty$, that means that the corresponding solutions of equation (1) tend towards a periodic solution whose amplitude and period do not depend on the initial conditions. Stationary motion established in mechanisms capable of self-oscillations always corresponds to limit cycles.

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[1] Summary of paper. Published in: VI Conference of Russian physicists, Moscow, Nizhnii-Novgorod, Kazan,Saratov, 5-16 August 1928, Gos. izd-vo, 1928, pp. 23-24.

Considerations connected with the behaviour of integral curves in the *xy* plane permit the application of qualitative theory to these processes, and allow certain general conclusions to be drawn concerning concrete physical problems. From this point of view, questions of ‘self-excitation’, ‘perturbations’ and ‘self-oscillation’ are considered. As examples of real systems a valve oscillator and other mechanisms are considered.

In conclusion, the method of approximations for the quantitative solution of the problem of self-oscillations is considered, and its relationship with the work of other authors who have studied particular cases of these oscillations is established.

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